

UNIVERSITY OF
ILLINOIS LIBRARY
AT URBANA-CHAMPAIGN
BOOKSTACKS

330
B385
No. 1094 COPY 2

STX



BEBR

FACULTY WORKING
PAPER NO. 1094

THE LIBRARY OF THE
FEB 21 1985
UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

Default Risk, Insurance, and the
Mortgage Contract Under Uncertainty

Jan K. Brueckner

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

BEBR

FACULTY WORKING PAPER NO. 1094

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

November, 1984

Default Risk, Insurance, and the
Mortgage Contract Under Uncertainty

Jan K. Brueckner, Associate Professor
Department of Economics

Abstract

This paper uses a two-period model to analyse the borrower's choice of an optimal time pattern of mortgage payments in a world where future house prices are uncertain. Since a decline in the value of the house can make the borrower's equity negative, leading to default on the mortgage, lenders in the model will require borrowers to purchase mortgage insurance. The premium on the insurance policy will depend on the riskiness of mortgage, which in turn depends on the magnitude of the first of the two mortgage payments. Mortgages with large (small) first payments will carry low (high) insurance premiums. Taking this fact into account borrowers decide on the optimal riskiness of their mortgage. Borrowers who discount the future heavily choose risky mortgages, while those who place a higher value on future consumption opt for less risky contracts.



Digitized by the Internet Archive
in 2011 with funding from
University of Illinois Urbana-Champaign

Default Risk, Insurance, and the
Mortgage Contract Under Uncertainty

by

Jan K. Brueckner

The private mortgage insurance industry, which exists to protect lenders from default losses on risky mortgages, has experienced rapid growth since 1970. Privately insured loans rose from 5.1% of total conventional lending in 1970 to 18.2% in 1980, with 70% of 1980 mortgages with loan-to-value ratios above 80% carrying private insurance.¹ Although growth of the private industry has made mortgage insurance more readily accessible to borrowers, the housing finance literature lacks an explicit treatment of the choice problem posed by the availability of insurance. The problem is a simple one: currently, the consumer can opt for a mortgage with a low loan-to-value ratio (no higher than 80%) and avoid the cost of insurance, or he can demand a riskier mortgage (a 90% loan, for example) and pay the corresponding insurance premiums. The necessity of insuring a risky mortgage makes it costly for the consumer to enjoy the postponement of housing expenses afforded by such a loan. However, a strong preference for present as opposed to future consumption may make bearing the cost of insurance worthwhile. The purpose of the present paper is to illustrate this basic proposition in a two-period model of mortgage choice that formalizes the trade-off between mortgage riskiness and insurance cost. The analysis explicitly treats the default behavior of borrowers, so that the lender's expected loss (and hence the required insurance premium) can be computed for any mortgage contract.² Since the model is

highly stylized, its implications are of little practical value. However, by providing the first detailed analysis of consumer incentives in mortgage markets with insurance, the paper fills a distinct gap in the housing finance literature.

The present analysis, which is predicated on the assumption that future house prices are uncertain, extends Brueckner's (1984) treatment of the optimal mortgage problem under certainty.³ In a certain world, mortgage insurance plays no role since default risk can be eliminated via the simple requirement that mortgage payments be high enough to avoid negative amortization. When house prices are constant or rising over time, this requirement guarantees that the borrower's indebtedness never rises above the value of the house, removing any incentive for default.⁴ With future house prices uncertain, however, avoidance of negative amortization will not eliminate default risk. The reason is that a sufficiently sharp decline in prices can always make the borrower's equity negative, creating a default incentive. With default risk inescapable under uncertainty, lenders will require that potential default losses be covered through insurance or some other mechanism in order to willingly extend loans.

With the requirement of insurance, the problem of choosing an optimal time path of mortgage payments assumes a different character than in the certainty case. The key difference is that even when negative amortization is avoided, a reduction in early payments increases the riskiness of the mortgage and consequently the size of the required insurance premium. As a result, the consumer bears a cost in postponing his housing expenses that is not present in a world of certainty.

To begin the analysis of the impact of insurance, the next section of the paper introduces the basic two-period model and analyzes the optimal mortgage under certainty. As well as generating results analogous to those of Brueckner (1984) for a discrete-time model, the certainty analysis provides a benchmark for the subsequent discussion. Section 3 adds uncertainty to the basic framework and considers the optimal mortgage problem under several different scenarios. Section 4 offers conclusions.

Before proceeding with the analysis, it is important to note the relationship between the present work and earlier papers dealing with the economics of collateral. Barro (1976), Benjamin (1978), Harris (1978), and Smith (1980, 1982) recognize the importance of the initial loan-to-value ratio in determining the probability of default when future asset value is random.⁵ However, rather than compensating the lender for default risk by insurance, compensation occurs via the contract interest rate in their models.⁶ A risky loan will require a high interest rate to be attractive to the lender. While this approach may help explain the general level of mortgage interest rates in the economy, it is not realistic when applied to individual mortgage contracts. For example, borrowers desiring relatively risky 90 percent loans are not quoted a higher interest rate in the existing market but rather are offered another avenue (i.e., insurance) for satisfying the lender's default concerns. Although the net result in both cases is to raise the level of house payments, the two approaches are conceptually and analytically distinct. By analysing the insurance approach to the problem of default risk, the present paper complements the other literature.

2. The Basic Framework and the Certainty Solution

In the model, consumers live for two periods, denoted zero and one. At the beginning of each period, income is received and housing costs are incurred. Income not spent on housing is saved until the end of the period, when purchases of a composite non-housing commodity are made. If the consumer still owns his house at the end of period one, the house is sold and the proceeds are used for non-housing consumption (there is no bequest motive in the model). The artifice of introducing a temporal distinction between the beginning and end of a period, which follows Schwab (1982), increases the versatility of a simple two-period framework.

In order to focus on issues of interest, housing consumption is fixed exogenously in the analysis. The purchase price of the fixed-size house is equal to v in period zero and is assumed to remain at v in period one under certainty (under uncertainty, the period-one price is random). In both cases, general inflation is absent in that the price of the non-housing commodity remains constant over time at a value of unity. The mortgage payments in the two periods are denoted m_0 and m_1 , and these payments must satisfy

$$m_0 + m_1/(1+r) = v, \quad (1)$$

where r is the one-period interest rate. Since the time pattern of mortgage payments is chosen by the consumer in the model, no down payment requirement is imposed (any down payment is simply part of m_0). Letting y_0 and y_1 denote incomes in the two periods and recalling that

income net of housing costs is saved until the end of each period, non-housing consumption equals

$$c_0 \equiv (y_0 - m_0)(1+r) \quad (2)$$

in period zero and

$$c_1 \equiv (y_1 - m_1)(1+r) + v \quad (3)$$

in period one for a consumer who holds his mortgage to maturity (recall that the house is sold at the end of period one in this case). It should be noted in (2) and (3) that the option of interperiod (as opposed to intraperiod) saving has been denied to the consumer (under this option, c_0 would be reduced by a savings term w_0 , with c_1 increased by $(1+r)w_0$). The reason is that such saving has no effect on the optimal mortgage under the utility function assumption imposed below. Unsecured borrowing against future income, which would involve a negative w_0 , is also ruled out (the reason, of course, is that the lender has no recourse in the event of default on such a loan).

A consumer who terminates his mortgage at the beginning of period one is assumed to enter the rental market, where the fixed-size house commands a rent of R per period.⁷ Under the assumption that houses last forever, R must satisfy $v = \sum_{i=0}^{\infty} R(1+r)^{-i}$, or $R = rv/(1+r)$. When the mortgage is terminated, the consumer follows one of two courses of action. If equity at the beginning of period one is positive, sale of the house and settlement of the mortgage is the mode of termination. When equity is negative, however, mortgage termination will occur via default (the house is abandoned without penalty to the consumer in this

case). Since equity at the beginning of period one is equal to the value of the house minus the balance due on the mortgage, or $v-m_1$, period-one consumption in the event that the mortgage is terminated equals

$$\{y_1 - rv/(1+r) + \max[v-m_1, 0]\}(1+r) \quad (4)$$

(the second term represents the rental payment). Note that the consumer enjoys positive proceeds from mortgage termination when equity is positive but escapes full repayment of his debt under default.⁸ Note also that in contrast to (3), (4) does not include a term representing proceeds from an end-of-period house sale.

The next step is to determine under what circumstances mortgage termination will occur. Termination is a desirable course of action for the consumer when the period-one consumption level given by (4) exceeds period-one consumption without termination, given by (3). Subtracting (3) from (4) yields

$$\{\max[v-m_1, 0] - (v-m_1)\}(1+r). \quad (5)$$

When $v-m_1 \geq 0$, (5) equals zero, indicating that a mortgage termination involving sale of the house results in the same level of non-housing consumption as holding the mortgage to maturity. When $v-m_1 < 0$, however, (5) reduces to $(1+r)(m_1-v) > 0$, so that termination (which in this case involves default) yields a higher consumption level.

The preceding discussion establishes the natural proposition that the borrower will default in period one when equity is negative. Since the lender loses money in the event of default (his loss equals

$(m_1 - v)/(1+r)$ in present value terms when foreclosure costs are zero), the mortgage contract will be structured to prevent its occurrence. In particular, the mortgage payment stream must satisfy $m_1 \leq v$, or equivalently $m_0 \geq rv/(1+r)$, using (1). The latter inequality states that the first mortgage payment must at least cover interest on the original loan balance (this requirement was referred to as the "default risk constraint" in Brueckner (1984)). When the default risk constraint is satisfied, default will never occur in a world of certainty.

The last step in the treatment of the certainty case is analysis of the optimal mortgage contract from the consumer's point of view. Although this is easily done using a general utility function, the analysis of the uncertainty case in Section 3 is intractable without a strong assumption on the form of the function. The present analysis will proceed under this assumption, which requires that the utility function has the risk-neutral form $u(c_0, c_1) \equiv c_0 + c_1/(1+\theta)$, where the c_1 are non-housing consumption levels in the two periods and $\theta > 0$ is the subjective discount rate. Note that since housing consumption is fixed, it need not appear in the utility function. Substituting (2) and (3) into the above function and eliminating m_0 using (1), utility equals

$$(1+r)(y_0 + y_1/(1+\theta)) - \frac{\theta+r(1+\theta)}{1+\theta} v + \frac{\theta-r}{1+\theta} m_1 \quad (6)$$

The consumer's problem is to maximize this expression by choice of m_1 subject to the constraint $m_1 \leq v$. The solution is immediate: when $\theta > r$, the optimal mortgage has $m_1 = v$ and $m_0 = rv/(1+r)$ (the

mortgage is a 100% loan with interest only paid in period zero); when $\theta < r$, a mortgage with $m_1 = 0$ and $m_0 = v$ is optimal (note that the house is actually purchased outright); when $\theta = r$, all feasible mortgages are equally attractive.⁹ The intuition behind these results is straightforward. When θ is high, the future is lowly valued and it is optimal to skew consumption toward period zero via the highest possible m_1 . When θ is low, future consumption is highly valued and it is optimal to pay for the house entirely in period zero, allowing the largest possible non-housing consumption in period one.

With the certainty solution as a benchmark, the analysis now turns to the uncertainty case, where default risk is pervasive and mortgage insurance plays a key role.

3. The Mortgage Contract under Uncertainty

The analysis of uncertainty will proceed under three alternative sets of assumptions. Section 3.1 considers the simplest possible model, where uncertainty over the period-one house value is added to the basic framework of Section 2. Use of the linear utility function makes the results of this analysis similar to those of the certainty case, with one of two corner solutions being optimal. The subsequent discussion shows that interior optima are possible under modified versions of the model. Section 3.2 investigates the case where the lender incurs a foreclosure cost under default, and Section 3.3 explores the effect of the assumption that mortgage interest is tax deductible. As well as demonstrating the existence of interior solutions, the analysis in Sections 3.2 and 3.3 gives additional insight into the structure of the model.

3.1 Uncertainty in the Simplest Model

Under uncertainty, the period-one house value is a random variable $\tilde{v}$ rather than a fixed quantity. The expected value of $\tilde{v}$ is assumed to equal v , the period-zero house value, and $\tilde{v}$'s density function $f(\tilde{v})$ is assumed to be differentiable, single-peaked, and symmetric. The minimum and maximum house values are denoted a and b , respectively (the assumption $b > (1+r)v$ is imposed to avoid inessential complications). While the expected value assumption is not used directly in the analysis, it guarantees that housing investors earn zero expected profits (see footnote 15 below). The symmetry assumption on f implies that the mode of $\tilde{v}$ (the value at the peak of f) equals v , the expected value, a fact that will prove useful in the analysis.

As shown in Section 2, mortgage default will occur when the period-one house value is less than m_1 . The probability of default is therefore the probability that $\tilde{v}$ is less than m_1 , or $\int_a^{m_1} f(\tilde{v})d\tilde{v}$. Note that since this integral equals $-\int_a^{m_1} f(\tilde{v})d\tilde{v} = 0$ when $m_1 < a$, it gives the correct default probability (zero) even in the case where m_1 lies below the range of possible house values. When default occurs, the lender's loss in present value terms is $(m_1 - \tilde{v})/(1+r)$, and the expected present value of the loss is given by

$$x \equiv \frac{1}{1+r} \int_a^{m_1} (m_1 - \tilde{v})f d\tilde{v} \quad (7)$$

(note that (7) equals zero for $m_1 \leq a$).

In a competitive mortgage market, the prospect of earning an expected present value of profit equal to zero will induce the (risk-neutral) lender to offer any given mortgage. This means that to

secure a mortgage with a particular value of m_1 , the borrower must make an up-front payment to the lender equal to his expected loss, given by x in (7). Equivalently, the borrower can purchase a mortgage insurance policy that guarantees full compensation to the lender in the event of default. The premium on such a policy is given by (7). Note that since losses are fully covered under all possible mortgages, the default risk constraint of Section 2 plays no role. A borrower is free to choose an m_1 greater than v provided he pays the appropriate insurance premium. Note also that since

$$\frac{\partial x}{\partial m_1} = \frac{1}{1+r} \int_a^{m_1} f d\tilde{v} \quad (8)$$

from (7), the insurance premium increases with m_1 when $m_1 > a$ at a rate proportional to the default probability. The reason, of course, is that a higher m_1 makes the mortgage riskier, with higher expected losses.¹⁰

With the risk-neutral utility function introduced in Section 2, expected utility under uncertainty is equal to

$$(y_0 - m_0 - x)(1+r) + \frac{1}{1+\theta} \int_{m_1}^b [(y_1 - m_1)(1+r) + \tilde{v}] f d\tilde{v} \\ \frac{1}{1+\theta} \int_a^{m_1} [y_1 - r\tilde{v}/(1+r)](1+r) f d\tilde{v}. \quad (9)$$

Note that for $\tilde{v} < m_1$, period-one consumption reflects the occurrence of default (consumption equals $(1+r)$ times income less rent, which is random since house value is¹¹), while for $\tilde{v} > m_1$, the consumption term indicates that the mortgage is held to maturity. When $\tilde{v} = m_1$, of course, the two consumption terms are equal. Note also that period-

zero consumption is reduced by the amount of the mortgage insurance premium.

The objective function of the borrower is derived by substituting the insurance premium from (7) into the utility expression (9) while using (1) to eliminate m_0 . After simplifying and gathering terms, the resulting expression reduces to

$$\Omega + \frac{\theta-r}{1+\theta} \left[\int_a^{m_1} v f d v + m_1 \int_{m_1}^b f d v \right], \quad (10)$$

where Ω represents the first two terms in (6). Note that the expression in brackets in (10) is the expected period-one return to the lender. Differentiating (10) with respect to m_1 yields

$$\frac{\theta-r}{1+\theta} \int_{m_1}^b f d v. \quad (11)$$

Since (11) is positive when $\theta > r$, (10) is increasing in m_1 and the borrower chooses the riskiest possible mortgage, setting $m_1 = (1+r)v$ (the largest possible value) and $m_0 = 0$. Note that when $\theta > r$, maximizing utility requires maximizing the lender's expected period-one return. When $\theta < r$, (11) is negative and a totally riskless mortgage (with $m_1 = 0$, $m_0 = v$, and $x = 0$) is optimal. In this case, maximizing utility means minimizing the lender's expected period-one return. Finally, when $\theta = r$, the optimal mortgage is indeterminate.

These outcomes are ultimately due to the same incentives as those at work in the certainty case: a high (low) value of θ means that future consumption is lowly (highly) valued, causing the consumer to skew his consumption toward period zero (one) through the mortgage contract.

The similarity of outcomes, however, masks an important difference in the incentive structures of the two problems, namely that a higher m_1 carries the penalty of a higher insurance premium in the uncertainty model. Since this penalty reinforces the consumer's desire to minimize m_1 in the case where future consumption is highly valued ($\theta < r$), the outcome $m_1 = 0$ is understandable. What is not obvious a priori, however, is that in the case where future consumption is lowly valued ($\theta > r$), the premium penalty is not sufficient to deter the consumer from securing the highest possible period-zero consumption via the riskiest possible mortgage. In the next section, it will be shown that the presence of foreclosure costs increases the magnitude of the insurance premium sufficiently to temper the consumer's desire to postpone his housing costs, leading to an interior solution to the optimization problem in the case where $\theta > r$.¹²

Before considering the model with foreclosure costs, it will be helpful to illustrate the results of this section diagrammatically. In Figure 1, the heavy upward sloping curve illustrates the borrower's opportunity locus, which shows the relationship between the required insurance premium and m_1 . The locus is strictly convex over its positive range.¹³ The indifference curves of the consumer are linear for $m_1 < a$ and strictly convex above a . When $\theta > r$, the curves are upward sloping and always steeper than the opportunity locus. When $\theta < r$, the curves are downward sloping for $m_1 < a$, although their slope may increase sufficiently above a so that they slope up for large values of m_1 .¹⁴ Lower indifference curves have higher utility levels. Figure 1 shows families of indifference curves for the two cases, along with the corresponding optimal (m_1, x) pairs.¹⁵

3.2 The Model with Foreclosure Costs

Foreclosure is in reality a costly process involving significant administrative and legal expenses on the part of the lender. To incorporate such expenses in the model, the lender is assumed to bear a cost of $s > 0$ in the event of default. Since the lender will require that mortgage insurance cover this additional loss, the premium becomes

$$x = \frac{1}{1+r} \int_a^{m_1} (s + m_1 - \tilde{v}) f d\tilde{v}, \quad (12)$$

an expression that exceeds the earlier premium (7). Substituting (1) and (12) into (9) to derive the borrower's objective function with foreclosure costs yields

$$\Omega + \frac{\theta-r}{1+\theta} \left[\int_a^{m_1} \tilde{v} f d\tilde{v} + m_1 \int_{m_1}^b \tilde{v} f d\tilde{v} \right] - s \int_a^{m_1} f d\tilde{v}, \quad (13)$$

Note that (13) is identical to (10) except for the presence of the last term, which equals minus the expected foreclosure cost. The derivative of (13) with respect to m_1 is

$$\frac{\theta-r}{1+\theta} \int_{m_1}^b \tilde{v} f d\tilde{v} - s f(m_1). \quad (14)$$

When $\theta < r$, (14) is negative, implying that the optimal mortgage has $m_1 = 0$, as in the model of Section 3.1. When $\theta > r$, however, (14) may equal zero for some interior value of m_1 . Whether such a solution represents the global optimum depends on satisfaction of the second-order condition. Differentiating (14), concavity of the objective function requires

$$-\frac{\theta-r}{1+\theta} f(m_1) - s f'(m_1) \leq 0. \quad (15)$$

Condition (15) is satisfied as an equality for $m_1 \leq a$ (where $f = f' = 0$) and as a strict inequality for $a < m_1 \leq v$ since $f > 0$ and $f' > 0$ hold in this range (recall that v is the mode of f).¹⁶ Since the density slopes downhill beyond v , however, (15) need not hold for m_1 between v and $(1+r)v$. In order to avoid the complications introduced by this possibility, the following discussion will assume that (15) is satisfied as a strict inequality for all m_1 between a and $(1+r)v$. Note that when f slopes gently near its peak, so that f' is never far from zero, this assumption will be especially likely to hold.

With (15) satisfied, location of the optimum is straightforward. If (14) equals zero at some interior m_1 , that solution represents the global optimum.¹⁷ If no interior solution exists, then the corner solution $m_1 = (1+r)v$ is optimal. To find the circumstances leading to an interior solution, note first that for $m_1 \leq a$, (14) equals the positive quantity $(\theta-r)/(1+\theta)$, ruling out a corner solution with $m_1 = 0$ or an interior solution in the interval $(0, a]$.¹⁸ As m_1 rises above a , $f(m_1)$ rises above zero and $\int_a^b f d\tilde{v}$ falls below unity, leading to a decline in (14). Using Figure 2,¹⁹ which shows a graph of the density function f , it is possible to derive a simple inequality which insures that this decline is sufficiently rapid to reduce (14) to zero before m_1 reaches v , guaranteeing the existence of an interior optimum below v . The inequality comes from noting in Figure 2 that the area of the box with its corner at the peak of f is unambiguously greater than the shaded area under f to the right of v . In other words, $vf(v) > \int_v^b f dv$. Note, however, that if s satisfies $(1+\theta)s/(\theta-r) \geq v$, then $(1+\theta)sf(v)/(\theta-r) > \int_v^b f d\tilde{v}$ will also hold. But this last inequality

means that (14) evaluated at v is negative, implying that the expression equals zero somewhere below v . A sufficient condition for existence of an interior optimum below v is therefore

$$s \geq (\theta-r)v/(1+\theta). \quad (15)$$

Thus, if the foreclosure cost exceeds some fraction of the period-zero house value (note $(\theta-r)/(1+\theta) < 1$), it will be optimal for the borrower to modify the extreme behavior of Section 3.1, choosing a mortgage of moderate instead of maximal riskiness.

Eq. (15) overstates the magnitude of s required for an interior solution by a wide margin. The minimal requirement for an interior optimum is simply that (14) evaluated at $m_1 = (1+r)v$ is negative, which translates into the condition

$$s > \frac{\theta-r}{1+\theta} \left[\int_{(1+r)v}^b \tilde{f} dv / f((1+r)v) \right]. \quad (16)$$

Inspection of Figure 2 shows that the ratio of the integral to the height of the density in (16) will typically be much less than v , implying that condition (16) is less stringent than (15). When s fails to satisfy this weaker condition, the increment to the insurance premium resulting from foreclosure costs is not sufficient to moderate the consumer's desire to postpone his housing costs, and a corner solution involving the riskiest possible mortgage is once again optimal.

Figure 3 illustrates an interior optimum. Indifference curves have the same slope and curvature as before, while the opportunity locus is steeper. The opportunity locus is strictly convex between a and v ,

although its curvature between v and $(1+r)v$ is ambiguous (the diagram shows convexity in this range).¹⁹

The value of m_1 at an interior optimum depends on the levels of the parameters s , θ , and r . The nature of this dependence is summarized by the following comparative static derivatives, which are derived by totally differentiating (14):

$$\frac{\partial m_1}{\partial s} = \Lambda f(m_1) < 0 \quad (17)$$

$$\frac{\partial m_1}{\partial \theta} = - \frac{(1+r)\Lambda}{1+\theta} \int_{m_1}^b \tilde{f} dv > 0 \quad (18)$$

$$\frac{\partial m_1}{\partial r} = \frac{\Lambda}{1+\theta} \int_{m_1}^b \tilde{f} dv < 0 \quad (19)$$

(Λ is the reciprocal of the negative expression in (15)). Eq. (17) shows that by increasing the magnitude of the insurance premium, and hence the penalty associated with a risky mortgage, an increase in s reduces the mortgage's optimal riskiness, lowering m_1 and x and raising m_0 . Eq. (18) shows that by reducing the value of already lowly-valued future consumption, an increase in θ makes a greater transfer of purchasing power toward period zero optimal, raising m_1 and x and reducing m_0 . An increase in r alters the trade-offs perceived by both the lender and borrower (the slopes of both the indifference curves and opportunity locus fall). The net effect is to reduce m_1 , as shown in (19) (x falls and m_0 increases).

As an additional comparative static exercise, it is interesting to derive the effect on the optimal m_1 of a shift in the distribution of period-one house values. Rewriting $f(\tilde{v})$ as $f(\tilde{v}-\alpha)$, where α is a shift

parameter, a shape-preserving rightward shift of the density function corresponds to an increase in α . Noting in (14) that the upper limit of integration must be replaced by $b+\alpha$, total differentiation yields the comparative static derivative²⁰

$$\frac{\partial m_1}{\partial \alpha} = 1, \quad (20)$$

indicating that a rightward shift in the density raises m_1 by exactly the amount of the shift. The reason for this result is that the shift in f makes any given mortgage less risky, reducing x and enticing the consumer to increase m_1 .

Finally, it is interesting to consider a solution to the optimization problem based on a particular form for the density f . When f is uniform on the interval $(v - \epsilon, v + \epsilon)$, so that $\tilde{f}(\tilde{v}) = 1/2\epsilon$ in this range,²¹ (14) yields

$$m_1 = v + \epsilon - \frac{1+\theta}{\theta-r} s, \quad (21)$$

indicating that m_1 equals the maximum house value minus a multiple of s .²² While the above comparative static results can be verified by direct calculation, (21) yields the additional implication that m_1 rises as the variance of the uniform distribution increases (as ϵ rises). This does not appear to be a general result.^{23,24}

3.3 The Model with Interest Deductibility

The previous section showed that when default imposes a resource cost on the lender, the solution to the optimization problem may involve a mortgage of intermediate riskiness requiring a moderate insurance

premium. When the assumption that mortgage interest is tax deductible is added to the basic framework of Section 3.1, an interior solution may emerge for somewhat different reasons. Under this modification, the borrower becomes concerned with generating a large interest deduction, and a particular trade-off assumes a key role. The trade-off is a result of the fact that while a risky mortgage generates a large deduction as a result of its low m_0 , the likelihood of default (which results in loss of the deduction) is also high. These opposing effects cause the borrower to gravitate toward a mortgage of intermediate riskiness, drawing him away from the corner solutions of Section 3.1. In order to illustrate this point most clearly, the analysis will first consider the case where $\theta = r$, in which the optimal mortgage was previously indeterminate.

Under the assumption that the income tax rate is a constant $t > 0$ and that interest income is nontaxable,²⁵ taxes in the two periods are $t(y_i - I_i)$, $i = 0, 1$, where the I_i represent mortgage interest. Since mortgage interest is equal to the balance due on the mortgage times the factor $r/(1+r)$, it follows that $I_0 = rv/(1+r)$ and $I_1 = rm_1/(1+r)$. Repeating the argument of Section 2, mortgage default in the present model occurs when $\tilde{v} < m_1 - tI_1$ (since the interest deduction is lost when default occurs, $\tilde{v}$ must be below m_1 by at least the amount of the deduction for default to be desirable). Substituting for I_1 , default requires $\tilde{v} < \frac{1+(1-t)r}{1+r} m_1 \equiv \delta m_1$ (note $\delta < 1$).

The mortgage insurance premium now becomes

$$x = \frac{1}{1+r} \int_a^{\delta m_1} (m_1 - \tilde{v}) f d\tilde{v} \quad (22)$$

and expected utility is given by

$$((1-t)y_0 + trv/(1+r) - x - m_0)(1+r) \quad (23)$$

$$+ \frac{1}{1+\theta} \int_{\delta m_1}^b \{[(1-t)y_1 - \delta m_1](1+r) + \tilde{v}\} f d\tilde{v} \\ + \frac{1}{1+\theta} \int_a^{\delta m_1} [(1-t)y_1 - rv/(1+r)](1+r) f d\tilde{v}$$

(note that period-one consumption without default is

$(1-t)y_1 - m_1 + trm_1/(1+r) = (1-t)y_1 - \delta m_1$). Substituting (1) and (22) into (23) and simplifying, the borrower's objective function becomes

$$(1-t)(1+r)(y_0 + y_1/(1+\theta)) - \frac{(1-t)r+\theta(1+(1-t)r)}{1+\theta} v \\ + \frac{\theta-r}{1+\theta} \int_a^{\delta m_1} \tilde{v} f d\tilde{v} + m_1(1 - \frac{(1+r)\delta}{1+\theta}) \int_{\delta m_1}^b f d\tilde{v}. \quad (24)$$

Setting $\theta = r$ to consider the simplest case, the last two terms of (24) reduce to

$$\frac{trm_1}{1+r} \int_{\delta m_1}^b \tilde{v} f d\tilde{v}. \quad (25)$$

Since (25) equals the expected period-one interest deduction (the magnitude of the deduction times the probability of no default), the borrower's goal in maximizing utility is simply to choose m_1 to maximize the expected deduction. The trade-off noted above is clear in (25): a higher m_1 increases the deduction but, by shrinking the value of integral, reduces the probability that it will be enjoyed.

The first-order condition for maximizing (25) reduces to

$$\int_{\delta m_1}^b \tilde{v} f d\tilde{v} - \delta m_1 f(\delta m_1) = 0. \quad (26)$$

Using the previous diagrammatic argument based on Figure 2, the LHS of (26) will be negative for $\delta m_1 = v$. Therefore, (26) is guaranteed to have a solution below $m_1 = v/\delta = (1+r)v/(1+(1-t)r) < (1+r)v$. When the objective function is strictly concave in the relevant range, this interior solution will represent the global optimum.²⁶ Thus, the desire to maximize the expected interest deduction causes the consumer to choose a mortgage of intermediate riskiness, with m_1 below $(1+r)v$ and m_0 above zero. Recall that in the absence of interest deductibility, the borrower was indifferent to the features of the mortgage contract in the $\theta = r$ case.²⁷

A similar effect emerges when $\theta \neq r$. Differentiating (24), the first-order condition for a utility maximum is

$$\frac{\theta-r+tr}{1+\theta} \int_{\delta m_1}^b \tilde{f} dv - \frac{tr}{1+r} \delta m_1 f(\delta m_1) = 0. \quad (27)$$

When $\theta > r$, a solution to (27) satisfying $m_1 < (1+r)v$ may exist and represent the optimum (subject to the usual qualifications).²⁸

Interest deductibility may therefore draw a borrower with $\theta > r$ (who preferred the riskiest possible mortgage in Section 3.1) toward a mortgage of intermediate riskiness. What is noteworthy about (27), however, is that a borrower who found a riskless mortgage optimal before (someone with $\theta > r$) may now prefer a risky contract. This follows because the first term in (27) is positive (and an interior solution is possible) as long as $\theta > (1-t)r$. As a result, borrowers with θ 's satisfying $(1-t)r < \theta < r$, who previously chose mortgages with $m_1 = 0$, may now find a positive value of m_1 optimal. Thus, mortgage

interest deductibility draws borrowers away from both corner solutions of Section 3.1 as they adopt contracts with high expected deductions.^{29,30}

4. Conclusion

This paper has analyzed the consumer's choice of an optimal mortgage contract in the face of a market trade-off between mortgage riskiness and the cost of insurance. The analysis in effect offers a theory of the "demand" for mortgage insurance, with borrower time preferences playing a key role. The theory shows that consumers who place a low value on future consumption demand risky mortgages and hence large insurance policies, while borrowers who value the future more highly opt for less risky mortgages carrying low insurance premiums. The risk-neutral utility function used in the analysis led, of course, to rather extreme forms of such behavior, with borrowers choosing either riskless or maximally risky mortgages in the basic model of Section 3.1. The subsequent analysis showed, however, that mortgages of intermediate riskiness (and moderate demands for insurance) can be optimal when modifications such as foreclosure costs or interest deductibility are added to the model.

An important goal for future research would be to construct a model of mortgage insurance based on a more realistic portrayal of the consumer's incentive to default on a mortgage contract. While the paper's assumption that default is costless to the borrower can be modified by introducing an exogenous default cost (see footnote 23), this is an inadequate way of capturing the powerful incentives against default that exist in the real world. Since these incentives mainly hinge on impairment of future borrowing power, a formal treatment

would require extension of the borrower's time horizon beyond a simple two-period framework. Construction of such a model would no doubt provide important additional insights.

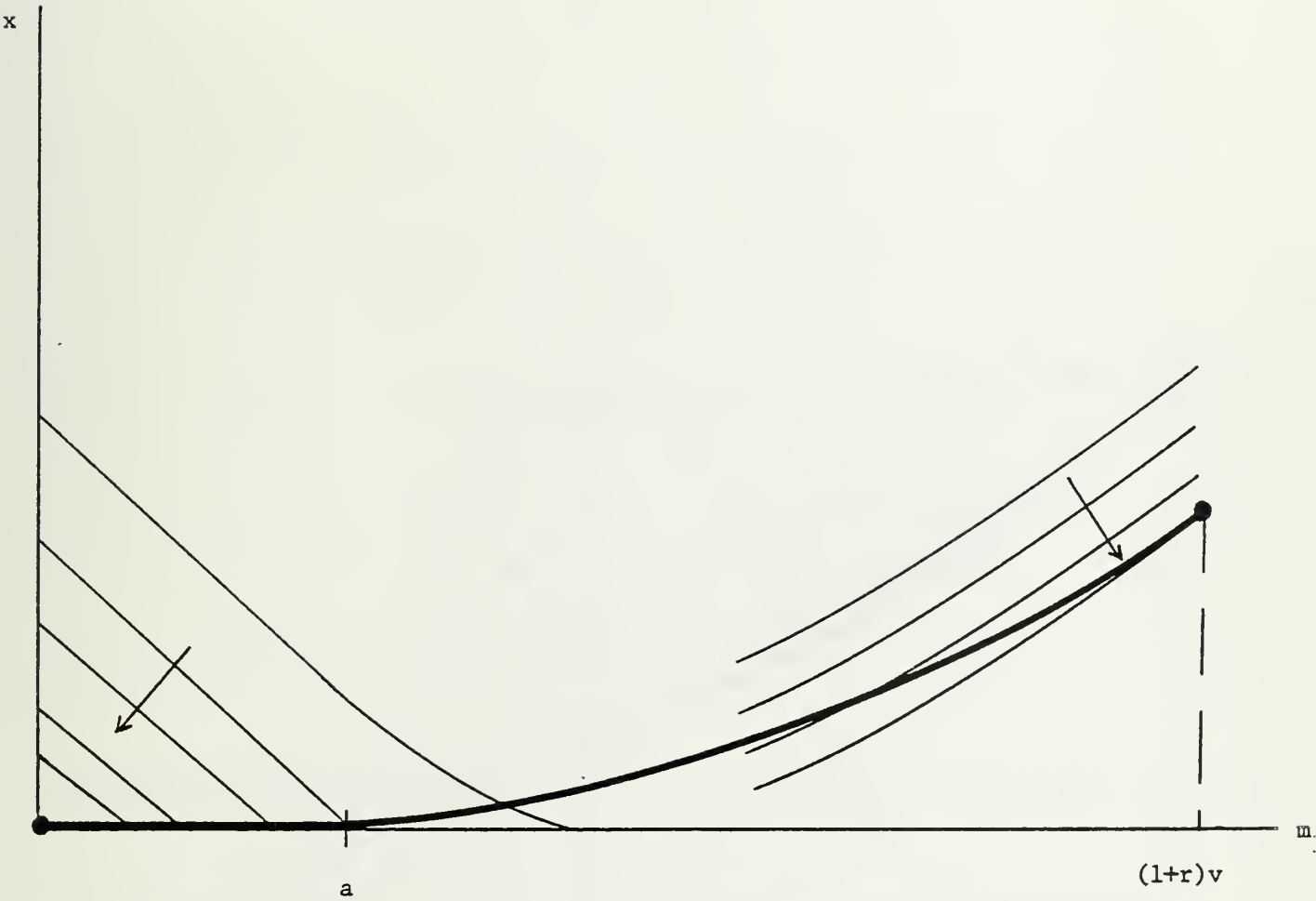


FIGURE 1.

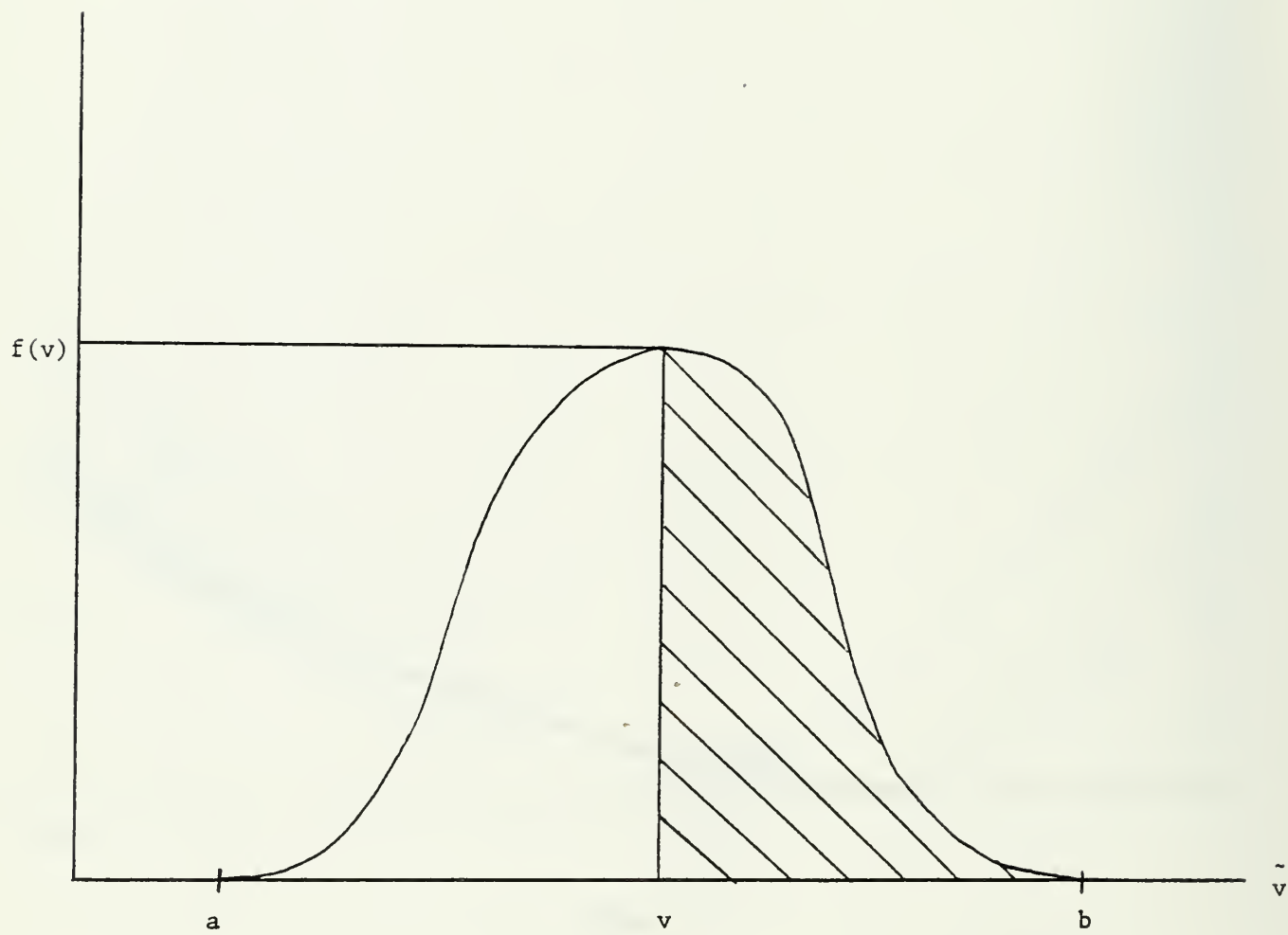


FIGURE 2.

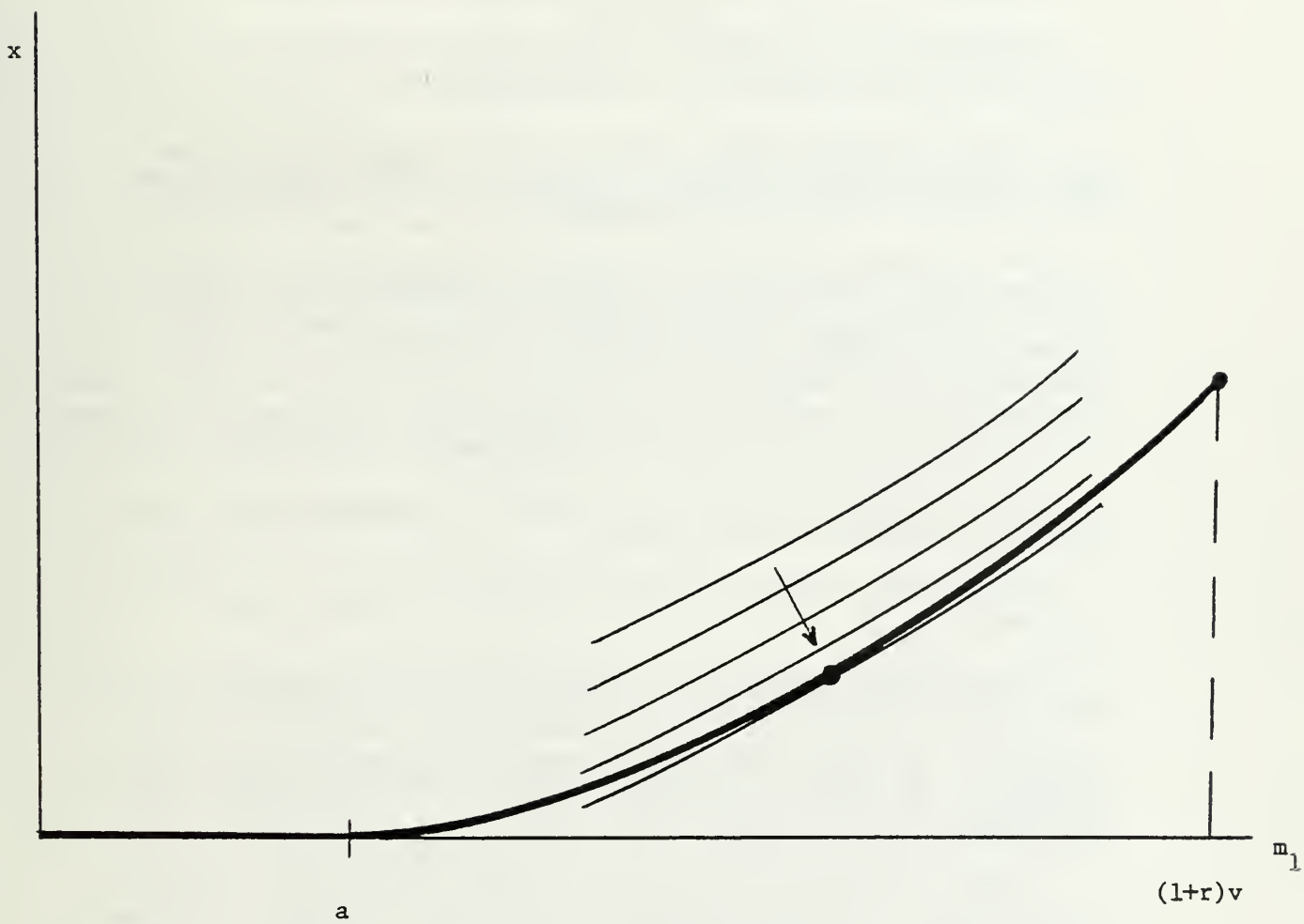


FIGURE 3.

Footnotes

¹These data are taken from Swan (1982).

²Vandell (1978) and Campbell and Dietrich (1982) present empirical evidence on the determinants of mortgage default.

³Alm and Follain (1984) present a related simulation analysis investigating consumer response to graduated payment mortgages and other alternative mortgage instruments.

⁴For a consumer who places a high value on present consumption, Brueckner (1984) showed that the optimal mortgage payment stream consists of two segments. Over the early part of the loan's life, payments are constant over time and cover interest only. The second segment has rising payments covering both interest and principal. This time pattern of payments allows the consumer to delay his housing costs as long as possible without exposing the lender to the risk of default.

⁵Since these models involve a single loan repayment, the time profile of payments is not an issue.

⁶Smith (1980) briefly considers mortgage insurance as a means of compensation.

⁷An equivalent assumption is that the consumer purchases outright a new house that is sold at the end of the period (see footnote 8).

⁸When a new house is purchased outright instead of rented (see footnote 7), period-one consumption is $(y_1 - v + \max[v - m_1, 0])(1+r) + v$, an expression that reduces to (4).

⁹It can be shown that utility level under the optimal mortgage is at least as high as that afforded by continuous renting, a relationship that also holds under uncertainty.

¹⁰In actual mortgage markets, insurance is not required when the loan-to-value ratio is 80 percent or less. It is interesting to note that in order for this outcome to emerge in the present stylized model, the m_1 associated with an 80 percent conventional loan must be exactly equal to a , the minimum house value (referring to (7), $m_1 = a$ gives $x = 0$). Using this fact, it is possible to calculate what the minimum house value must be in order for lenders to offer 80 percent loans without insurance. Since the constant principal and interest payment p under a conventional 80 percent mortgage must satisfy $p + p/(1+r) = .8v$, it follows that $m_1 = p = .8(1+r)v/(2+r)$ (note that $m_0 = .2v + p$). The requirement $m_1 = a$ therefore implies that the

period one house value is expected to be no smaller than 40 percent of v (note $(1+r)/(2+r) \approx .5$). While the lack of realism of the model means that this conclusion cannot be taken seriously, the calculation is nevertheless instructive.

¹¹The implicit assumption that house values are equal to the capitalized value of current rents requires myopic expectations on the part of investors.

¹²Strictly speaking, it is the increase in the slope of the relationship between x and m_1 (see Fig. 1 below), rather than the increase in its level, that leads to the interior solution.

¹³From (8), the second derivative is $f(m_1)/(1+r) \geq 0$.

¹⁴From (9), the slope of the indifference curves is

$$\frac{1}{1+r} \left[\int_a^{m_1} f d\tilde{v} + \frac{\theta-r}{1+\theta} \int_{m_1}^b f d\tilde{v} \right]$$

and the second-derivative is

$$f(m_1)/(1+\theta) \geq 0.$$

¹⁵In the model of Section 2, easy calculations show that housing investors earn zero profit, regardless of whether they are self-financing or use mortgage finance. While self-financing also yields zero expected profit in the uncertainty case as long as the expected value of $\tilde{v}$ equals v , it is important to verify that mortgage-based housing investment similarly earns zero expected profit under this assumption. The first step is to note that the default rule for a housing investor is identical to the one derived for consumers (period-one default occurs when equity is negative). This follows because period-one profit is $(r\tilde{v}/(1+r) - m_1)(1+r) + \tilde{v}$ when the mortgage is held to maturity and $(\max[\tilde{v} - m_1, 0])(1+r)$ otherwise. Using this result and noting that period-zero profit is $(rv/(1+r) - m_0 - x)(1+r)$, the expected present value of profit equals $rv + E\tilde{v} - (1+r)m_0 - m_1$, where E denotes expected value. Under the maintained assumption $E\tilde{v} = v$, this expression equals zero.

¹⁶Note that since f is assumed to be differentiable everywhere, f must be continuous at a (so that $f(a) = 0$) and $f'(a) = 0$ must hold.

¹⁷ A useful interpretation of the first-order condition comes from rewriting (14) as

$$\frac{f(m_1)}{\int_{m_1}^b f dv} = \frac{\theta-r}{(1+\theta)s},$$

which says that m_1 is optimal when the relative rate of decline of the no-default probability is proportional to the reciprocal of the fore-closure cost.

¹⁸ When $f(a) > 0$, (14) is discontinuous at $m_1 = a$. If the expression is negative at this point, a mortgage with $m_1 = a$ is optimal.

¹⁹ From (12), the slope of the opportunity locus is

$$\frac{1}{1+r} \left[\int_a^{m_1} f dv + sf(m_1) \right]$$

and the second derivative is

$$\frac{f(m_1) + sf'(m_1)}{1+r}$$

²⁰ Differentiating the modified eq. (14) with respect to α yields

$$\begin{aligned} \frac{\theta-r}{1+r} \left[f(b) - \int_{m_1}^{b+\alpha} f'(v-\alpha) dv \right] + sf'(m_1-\alpha) = \\ \frac{\theta-r}{1+r} \left[f(b) - f(v-\alpha) \Big|_{m_1}^{b+\alpha} \right] + sf'(m_1-\alpha). \end{aligned}$$

Evaluating this expression and setting $\alpha = 0$ yields a result equal to $-1/\Lambda$.

²¹ Note that the uniform distribution has no peak and does not satisfy the smoothness requirements used in the analysis (it is discontinuous at the maximum and minimum house values).

²² Note that when (15) holds, (21) is less than or equal to ε . Since $v \geq \varepsilon$ must hold for the lower support of f to be positive, the result that the interior solution is less than v when (15) holds is verified.

²³ A diagrammatic argument shows that when f is single-peaked, a reduction in the variance of the distribution has an ambiguous effect on the value of (14).

²⁴ It may be shown that another modification similar to the introduction of foreclosure costs also has the potential for yielding an interior solution to the optimization problem. This is the assumption that borrower bears an explicit cost k in period one when default occurs. Under this modification, default is optimal when $\tilde{v} < m_1 - k$ (the argument parallels that of Section 2). The insurance premium becomes

$$x = \frac{1}{1+r} \int_a^{m_1 - k} (m_1 - \tilde{v}) f d\tilde{v},$$

while expected utility equals (9) with m_1 in the limits of integration replaced by $m_1 - k$ and k subtracted off from period-one consumption under default. The first-order condition for utility maximization is

$$\frac{\theta - r}{1+r} \int_{m_1 - k}^b \tilde{v} f d\tilde{v} - k f(m_1 - k) = 0,$$

which may have an interior solution when $\theta > r$.

²⁵ Adding the assumption that interest income is taxable introduces extra complications without essentially changing the basic insights offered by the analysis in this section.

²⁶ While the second derivative of (25), which equals $-2f(\delta m_1) - \delta m_1 f'(\delta m_1)$, need not be negative for $\delta m_1 > v$, we assume negativity holds over this range.

²⁷ This result reemerges when $t = 0$ in (25).

²⁸ The second derivative of (24), which equals

$$-\frac{\theta - r + tr}{1+\theta} f(\delta m_1) - \frac{tr\delta}{1+r} [f(\delta m_1) + \delta m_1 f'(\delta m_1)]$$

need not be negative for $\delta m_1 > v$. As before, we assume negativity holds. No simple condition guaranteeing existence of an interior optimum emerges in this case. The requirement is that (27) is negative for $m_1 = (1+r)v$.

²⁹ While comparative static results for the general model are ambiguous, the intuitive result $\partial m_1 / \partial t > 0$ emerges in the case where $\theta = r$ (by making the interest deduction worth more, a higher t draws the consumer toward a riskier mortgage). Additional results for the $\theta = r$ case are $\partial m_1 / \partial r > 0$ for $t < 1/2$ and $0 < \partial m_1 / \partial \alpha < 1$, where α is the density function shift parameter.

³⁰ It is interesting to note that when the income tax is progressive rather than proportional, the condition for default is

$$\tilde{v} < m_1 - [T(y_1) - T(y_1 - rm_1/(1+r))],$$

where T is the tax schedule. Although the level of income has no effect on default propensities in the models analysed in the paper, the fact that the RHS above is decreasing in y_1 (the derivative is $T'(y_1 - rm_1/(1+r)) - T'(y_1)$, which is less than zero since $T'' > 0$) means that high-income borrowers have a lower default probability for any given m_1 .

References

- J. Alm and J. R. Follain, "Alternative Mortgage Instruments, the Tilt Problem, and Consumer Welfare," Journal of Financial and Quantitative Analysis 19, 113-126 (March 1984).
- D. K. Benjamin, "The Use of Collateral to Enforce Debt Contracts," Economic Inquiry 16, 333-359 (1978).
- R. J. Barro, "The Loan Market, Collateral, and Rates of Interest," Journal of Money, Credit and Banking 8, 439-456 (1976).
- J. K. Brueckner, "The Flexible Mortgage: Optimal Financing of a Consumer Durable," Journal of the American Real Estate and Urban Economics Association 12, 136-152 (1984).
- T. S. Campbell and J. K. Dietrich, "The Determinants of Default on Insured Conventional Mortgages," Journal of Finance 38, 1569-1581 (December 1983).
- R. Harris, "The Consequences of Costly Default," Economic Inquiry 8, 477-496 (1978).
- R. Schwab, "Inflation Expectations and the Demand for Housing," American Economic Review 72, 143-153 (1982).
- C. W. Smith, "On the Theory of Financial Contracting: The Personal Loan Market," Journal of Monetary Economics 6, 333-357 (1980).
- C. W. Smith, "Pricing Mortgage Originations," Journal of the American Real Estate and Urban Economics Association 10, 313-330 (1982).
- C. Swan, "Pricing Private Mortgage Insurance," Journal of the American Real Estate and Urban Economics Association 10, 276-296 (Fall 1982).
- K. Vandell, "Default Risk under Alternative Mortgage Instruments," Journal of Finance 33, 1279-1296 (1978).

HECKMAN
BINDERY INC.



JUN 95

Bound-To-Pleas[®] N. MANCHESTER,
INDIANA 46962

UNIVERSITY OF ILLINOIS-URBANA



3 0112 060296099